"Fixed Point" QCD Analysis of the CCFR Data on Deep Inelastic Neutrino-Nucleon Scattering

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Abstract

The results of LO *Fixed point* QCD (FP-QCD) analysis of the CCFR data for the nucleon structure function $xF_3(x,Q^2)$ are presented. The predictions of FP-QCD, in which $\alpha_s(Q^2)$ tends to a nonzero coupling constant α_0 as $Q^2 \to \infty$, are in good agreement with the data. The description of the data is even better than that in the case of LO QCD. The FP-QCD parameter α_0 is determined with a good accuracy: $\alpha_0 = 0.198 \pm 0.009$. Having in mind the recent QCD fits to the same data we conclude that unlike the high precision and large (x,Q^2) kinematic range of the CCFR data they cannot discriminate between QCD and FP-QCD predictions for $xF_3(x,Q^2)$.

1. Introduction.

The progress of perturbative Quantum Chromodynamics (QCD) in the description of the high energy physics of strong interactions is considerable. The QCD predictions are in good quantitative agreement with a great number of data on lepton-hadron and hadron-hadron processes in a large kinematic region (e.g. see reviews [1] and references therein). Despite of this success of QCD, we consider that it is useful and reasonable to put the question: Do the present data fully exclude the so-called *fixed point* (FP) theory models [2] ?

We remind that these models are not asymptotically free. The effective coupling constant $\alpha_s(Q^2)$ approaches for $Q^2 \to \infty$ a constant value $\alpha_0 \neq 0$ (the so-called fixed point at which the Callan- Symanzik β-function $\beta(\alpha_0) = 0$). Using the assumption that α_0 is small one can make predictions for the physical quantities in the high energy region, as well as in QCD, and confront them to the experimental data. Such a test of FP theory models has been made [3, 4] by using the data of deep inelastic lepton-nucleon experiments started by the SLAC-MIT group [5] at the end of the sixties and performed in seventies [6]. It was shown that

i) the predictions of the FP theory models with *scalar* and *non- colored (Abelian) vector* gluons *do not agree* with the data

ii) the data *cannot distinguish* between different forms of scaling violation predicted by QCD and the so-called *Fixed point* QCD (FP-QCD), a theory with *colored vector* gluons, in which the effective coupling constant $\alpha_s(Q^2)$ does not vanish when Q^2 tends to infinity.

We think there are two reasons to discuss again the predictions of FP-QCD. First of all, there is evidence from the non-perturbative lattice calculations [7] that the β-function in QCD vanishes at a nonzero coupling α_0 that is small. (We remind that the structure of the β-function can be studied only by non-perturbative methods.) Secondly, in the last years the accuracy and the kinematic region of deep inelastic scattering data became large enough, which makes us hope that discrimination between QCD and FP-QCD could be performed.

In this paper, we present a leading order *Fixed point* QCD analysis of the CCFR data [8]. They are most precise data for the structure function $xF_3(x, Q^2)$. This structure function is pure non-singlet and the results of analysis are independent of the assump-

tion on the shape of gluons. To analyze the data the method [9] of reconstruction of the structure functions from their Mellin moments is used. This method is based on the Jacobi - polynomial expansion [10] of the structure functions. In [11] this method has been already applied to the QCD analysis of the CCFR data.

2. Method and Results of Analysis.

Let us start with the basic formulas needed for our analysis.

The Mellin moments of the structure function $xF_3(x,Q^2)$ are defined as:

$$M_n^{NS}(Q^2) = \int_0^1 dx x^{n-2} xF_3(x,Q^2) \ , \tag{1}$$

where $n = 2, 3, 4, ...$.

In FP-QCD the Q^2 evolution of the non-singlet moments at large Q^2 is given by

$$M_n^{NS}(Q^2) = M_n^{NS}(Q_0^2) \left[\frac{Q_0^2}{Q^2}\right]^{\frac{1}{2}\gamma_n^{NS}(\alpha_0)} \ , \tag{2}$$

where the anomalous dimensions γ_n^{NS} are determined by its fixed point value

$$\gamma_n^{NS}(\alpha_0) = \frac{\alpha_0}{4\pi}\gamma_n^{(0)NS} + (\frac{\alpha_0}{4\pi})^2\gamma_n^{(1)NS} + ..., \tag{3}$$

and

$$\gamma_n^{(0)NS} = \frac{8}{3}[1 - \frac{2}{n(n+1)} + 4\sum_{j=2}^{n}\frac{1}{j}] \ . \tag{4}$$

The n dependence of $\gamma_n^{(0)NS}$, $\gamma_n^{(1)NS}$, etc. is exactly the same as in QCD. However, the Q^2 behaviour of the moments is different. In contrast to QCD, the Bjorken scaling for the moments of the structure functions is broken by powers in Q^2 .

In the LO approximation of FP-QCD we have for the moments of $xF_3(x,Q^2)$:

$$M_n^{NS}(Q^2) = M_n^{NS}(Q_0^2) \left[\frac{Q_0^2}{Q^2}\right]^{\frac{1}{2}d_n^{NS}} \ , \tag{5}$$

where

$$d_n^{NS} = \frac{\alpha_0}{4\pi}\gamma_n^{(0)NS} \tag{6}$$

and α_0 is a free parameter, to be determined from experiment.

Having in hand the moments (5) and following the method [9, 10], we can write the structure function xF_3 in the form:

$$xF_3^{N_{max}}(x,Q^2) = x^{\alpha}(1-x)^{\beta}\sum_{n=0}^{N_{max}}\Theta_n^{\alpha,\beta}(x)\sum_{j=0}^{n}c_j^{(n)}(\alpha,\beta)M_{j+2}^{NS}\left(Q^2\right) , \tag{7}$$

where $\Theta_n^{\alpha\beta}(x)$ is a set of Jacobi polynomials and $c_j^n(\alpha,\beta)$ are coefficients of the series of $\Theta_n^{\alpha,\beta}(x)$ in powers in x:

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)x^j. \tag{8}$$

N_{max}, α and β have to be chosen so as to achieve the fastest convergence of the series in the R.H.S. of Eq.(7) and to reconstruct xF_3 with the accuracy required. Following the results of [9] we use $\alpha = 0.12$, $\beta = 2.0$ and $N_{max} = 12$. These numbers guarantee accuracy better than 10^{-3} .

Finally we have to parametrize the structure function xF_3 at some fixed value of $Q^2 = Q_0^2$. Following [11], where analysis of the same data is done in the framework of QCD, we choose $xF_3(x,Q^2)$ in the simplest form:

$$xF_3(x,Q_0^2) = Ax^B(1-x)^C \ . \tag{9}$$

The parameters A, B and C in Eq. (9) and the FP-QCD parameter α_0 are free parameters which are determined by the fit to the data.

To avoid the influence of higher–twist effects and the target mass corrections, we have used only the experimental points in the plane (x,Q^2) with $10 < Q^2 \leq 501\ (GeV/c)^2$. This cut corresponds to the following x range: $0.015 \leq x \leq 0.65$.

The results of the fit are presented in Table 1. In all fits only statistical errors are taken into account. It is seen from the Table that the values of α_0 and $\chi^2_{d.f.}$ are not sensitive to the particular choice of Q_0^2 . This is an indication of the stability and the self-consistence of the method used.

The values of $\chi^2_{d.f.}$ presented in Table 1 are slightly smaller than those obtained in the LO QCD analysis [11] of the CCFR data and indicate a good description of the data. The values of the parameters A, B and C are in agreement with the results of [11].

Q_0^2 $(GeV/c)^2$	$\chi^2_{d.f.}$	α_0	A	B	C	GLS sum rule
3	82.2/61	.198±.009	6.50±.18	.768±.013	3.44±.04	2.539±.111
10	82.9/61	.198±.009	5.93±.15	.722±.012	3.56±.034	2.564±.106
20	83.5/61	.198±.009	5.62±.15	.696±.012	3.64±.032	2.580±.111
50	84.5/61	.198±.009	5.24±.14	.663±.012	3.73±.031	2.605±.115
100	85.3/61	.198±.009	4.96±.13	.638±.012	3.80±.029	2.626±.117

Table 1. The results of the LO FP-QCD fit to the CCFR xF_3 data for $f = 4$. $\chi^2_{d.f.}$ is the χ^2-parameter normalized to the degree of freedom $d.f.$.

Previous estimations [4] of the FP-QCD parameter α_0 based on the analysis of SLAC deep inelastic electron-proton data provide a large region for possible values of α_0 :

$$0.1 < \alpha_0 < 0.4 \ . \tag{10}$$

Now α_0 is determined from the CCFR data with a good accuracy in the above interval:

$$\alpha_0 = 0.198 \pm 0.009 \ . \tag{11}$$

The value of the Gross-Llewellyn Smith (GLS) sum rule has been calculated at different values of Q_0^2 as the first moment of $xF_3(x, Q_0^2)$

$$GLS(Q_0^2) = \int_0^1 \frac{dx}{x} A(Q_0^2) x^{B(Q_0^2)} (1-x)^{C(Q_0^2)} \tag{12}$$

with an accuracy about 4%. These values (see Table 1) are in good agreement with LO QCD results of [11].

3. Summary.

The CCFR deep inelastic nucleon scattering data have been analyzed in the framework of the *Fixed point* QCD. It was demonstrated that the data for the nucleon structure function $xF_3(x, Q^2)$ are in good agreement with the LO predictions of this theory model using the assumption that the *fixed point* coupling α_0 is small. In contrast to the results of the fits to the previous generations of deep inelastic lepton-nucleon experiments, the value of this constant was determined with a good accuracy: $\alpha_0 = 0.198 \pm 0.009$. This value of α_0 is consistent with the assumption that α_0 is small.

In conclusion, we find that the CCFR data, the most precise data on deep inelastic scattering at present, *do not eliminate* the FP-QCD and therefore other tests have to

be made in order to distinguish between QCD and FP-QCD.

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